INSTRUCTIONS TO CANDIDATES

1. This examination paper contains 5 short questions in Part I and 3 long questions in Part II. It comprises 9 printed pages.
2. Answer ALL the questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. The total marks for Part I is 40 and that for Part II is 60.
6. Only non-programmable and non-graphing calculators without remote communication function may be used.
7. A table of constants and mathematical formulae is attached.
PART I

This part of the examination paper contains five short-answer questions on pages 2 to 4. Answer ALL questions.

Question 1
Consider the box-like Gaussian surface $G$ as shown in Figure 1.

![Figure 1]

The bottom face is in the $xz$-plane, while the top face is in the horizontal plane passing through $y = 2.00$ m. Suppose $x_1 = 1.00$ m, $x_2 = 4.00$ m, $z_1 = 1.00$ m, and $z_2 = 3.00$ m, and $G$ lies in an electric field given by

$$\vec{E} = \left[(-6.00x + 10.0)\hat{i} - 3.00\hat{j} + \kappa\hat{k}\right] \text{ N/C},$$

with $x$ and $z$ in metres and $\kappa$ is a constant. If $G$ encloses a net charge of $-48.0e_0$ C, find $\kappa$. [8]

Question 2
(a) Consider a particle $P$ of mass $m$ and positive charge $q$ moving with velocity $\vec{v}$ in a uniform static magnetic field $\vec{B}$.

(i) Describe the magnitude and direction of the magnetic force that $\vec{B}$ exerts on $P$. [3]

(ii) Can $\vec{B}$ do work on $P$? Explain briefly. [2]

(b) A source injects an electron of speed $v = 1.5 \times 10^6$ m/s into a uniform magnetic field of magnitude $B = 1.0 \times 10^{-2}$ T. The velocity of the electron makes an angle $\theta = 30^\circ$ with the direction of the magnetic field. Find the distance $d$ from the point of injection at which the electron next crosses the field line that passes through the injection point. [3]
Question 3
Figures 2 and 3 show two circuits, each with $R = 14.0 \, \Omega$, $C = 6.20 \, \mu F$, $L = 56.0 \, \text{mH}$, and a battery having emf $E = 34.0 \, \text{V}$.

The switch in Figure 2 is kept in position $a$ for $2\tau_c$ s, while the switch in Figure 3 is kept in position $a$ for $2\tau_L$ s. The switches are then separately thrown to their respective position $b$’s. Here, $\tau_c$ and $\tau_L$ are the capacitive and inductive time constants respectively. Find
(a) $\tau_c$ and $\tau_L$. [2]
(b) the angular frequency of the resulting oscillations in each circuit. [1]
(c) the current in each circuit 1.20 ms after the switches are being thrown to their respective position $b$’s. [5]

Question 4
A series $R$-$L$-$C$ circuit (Figure 4) consists of a resistor $R = 100 \, \Omega$, an inductor $L = 0.15 \, \text{H}$, and a capacitor $C = 30 \, \mu \text{F}$ connected in series to an 120 V (rms) emf source. The frequency of the source is 60 Hz.

Calculate
(a) the current amplitude. [4]
(b) the phase angle $\phi$. [2]
(c) the average power loss. [2]
Question 5
A plane electromagnetic wave travelling in the positive direction of an $x$-axis in vacuum has electric field components $E_x = E_y = 0$ and

$$E_z = E_{\text{max}} \cos(\omega t - kx),$$

with $E_{\text{max}} = 2.0 \text{ V/m}$ and $\omega = \pi \times 10^{15} \text{ s}^{-1}$.

(a) Find $k$. 
(b) What is the magnetic field associated with the wave?
(c) Calculate the wave intensity.
(d) The wave uniformly illuminates a surface of area 2.0 m$^2$. If the surface totally absorbs the wave, calculate the radiation pressure and the magnitude of the corresponding force on the surface.

END OF PART I
PART II

This part of the examination paper contains three long questions on pages 5 to 9. Answer ALL questions.

Question 6(a)
Positive electric charge $Q$ is uniformly distributed along a non-conducting rod $R$ with length $L$, lying along the $x$-axis between $x = 0$ and $x = L$, as shown in Figure 5.

Consider a point $P$ that lies on a line, which goes through a point on the $xy$-plane with coordinates $x = L/2$ and $y = L/2$, and is parallel to the $z$-axis. $P$ is at a height $z$ above the $xy$-plane. Show that the electric potential $V$ at $P$ is given by

$$V(z) = \frac{Q}{4\pi\varepsilon_0 L} \ln \left( \frac{\sqrt{2L^2 + 4z^2} + L}{\sqrt{2L^2 + 4z^2} - L} \right).$$
Question 6(b)
Now, we have four such rods as $R$, that are arranged into a square as shown in Figure 6.

Find the electric field $\vec{E}$ at point $P$. Explain clearly how you obtain your answer. \[9\]

Question 6(c)
If an electron of mass $m$ is placed at the centre of the square and is then displaced a small distance $z$ along the vertical dashed line as shown in Figure 6 ($z \ll L$), determine its oscillating frequency. You may ignore gravity. \[4\]
**Question 7(a)**
Consider a positive point charge $q$ moving with velocity $\vec{v}$ as shown in Figure 7.

![Figure 7](image)

Describe the magnitude and direction of the magnetic field at point $P$ due to $q$. \[6\]

**Question 7(b)**
Hence, derive the infinitesimal magnetic field $d\vec{B}$ at point $P$ due to a current element $ldl$ as shown in Figure 8.

![Figure 8](image)

**Question 7(c)**
Derive an expression for the magnetic field strength at distance $d$ from the centre of a straight wire of finite length $L$ that carries current $I$. You have to provide a diagram of the straight wire with clear labels of all the quantities you use in your derivation. \[8\]

**Question 8(a)**
(i) A wire with resistivity $\rho$ carries current $i_c$. The current is increasing at the rate $di_c/dt$. Show that there is a displacement current $i_d$ in the wire equal to $\rho \varepsilon_0 di_c/dt$. \[4\]

(ii) Consider a long, straight silver wire with resistivity $\rho = 1.62 \times 10^{-8} \Omega \cdot m$. The current in the wire is uniform and changing at the rate of 2000 A/s when the current is 100 A. What is the ratio of the magnitude of the magnetic field due to the displacement current to that due to the current at a small distance $r$ from the wire? \[2\]
Question 8(b)
The square loop in Figure 9 is made of rigid conducting rods, each 3.00 m in length, with a total series resistance of 10.0 Ω. It is placed in a uniform 0.100 T magnetic field directed perpendicularly into the plane of the paper.

The loop, which is hinged at each corner, is pulled until the separation between points $b$ and $d$ is 3.00 m as shown in Figure 10.
This process takes 0.100 s to be completed, and is executed in such a way that the rate of change in area bounded by $abcd$ is a constant.

(i) What is the direction of the induced current in the loop? Justify your answer.  
(ii) What is the power delivered to the resistors?
(iii) Using

$$E_{\text{ind}} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l},$$

show that the induced emf $E_{\text{ind}}$ in the loop at any instant during the 0.100 s is given by

$$E_{\text{ind}} = -1.80 \cos 20 \frac{d\theta}{dt},$$

where $\theta$ is the angle that $bc$ makes with the horizontal through $c$.  

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END OF PART II
END OF PAPER